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Vertical stabilization of tokamak plasmas via extremum seeking

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A B S T R A C T

In this paper we propose a vertical stabilization (VS) control system for tokamak plasmas based on the extremum seeking (ES) algorithm. The gist of the proposed strategy is to inject an oscillating term in the control action and exploit a modified ES algorithm in order to bring to zero the *average* motion of the plasma along the unstable mode. In this way, the stabilization of the unstable vertical dynamic of the plasma is achieved. The approach is validated by means of both linear and nonlinear simulations of the overall ITER tokamak magnetic control system, with the aim of demonstrating robust operation throughout the flat-top phase of a discharge and the capability of reacting to a variety of disturbances.

1. Introduction

Nuclear fusion is foreseen as a possible source of energy for the next century (American Physical Society Division Plasma Physics, 2020; EUROfusion, 2018). A big effort has been made, since the end of World War II, with the aim of developing its peaceful use towards the realization of a power plant. The tokamak concept arose in the 50s–60s of the last century, as one of the most promising experimental devices, aimed at proving the feasibility of energy production by means of nuclear fusion on Earth (Wesson & Campbell, 2011).

Since the mid 70s, many international projects have been successfully established to build and operate tokamaks all around the world. The JET tokamak (Wesson, 2000) is still the world's largest, although it will be soon exceeded in size by the joint EU-Japan project JT-60SA (Barabaschi et al., 2019) and by ITER (0000), which is an international enterprise involving EU, India, People's Republic of China, Russia, South Korea and USA, and which is currently under construction in France.

In a tokamak, a fully ionized gas of hydrogen ions, called *plasma*, is confined by magnetic fields and heated to temperatures of tens to hundreds millions degrees. At such high temperatures, collisions between ions can overcome the Coulomb repulsive forces, resulting into fusion reactions. Plasma confinement is achieved by means of both toroidal and poloidal magnetic field sources. In particular, the toroidal field component is produced by a set of coils wrapped around the vacuum vessel (see the blue coils in Fig. 1), while the poloidal one is generated by the presence of a plasma current induced in the ionized gas, and by a set of toroidally continuous coils (in grey in Fig. 1), called Poloidal Field (PF) coils.

Operation of large tokamaks such as ITER (Gribov et al., 2007) or future power plants such as DEMO (Ambrosino et al., 2021; Biel et al., 2022) calls for the solution of several challenging control problems, among which there is the so called *magnetic control problem*, i.e. the control of the current induced into the plasma, as well as of its shape and position, by regulating the poloidal field produced by the currents flowing in the PF coils (Ariola & Pironti, 2016). Accurate plasma position and shape control is needed for several reasons: from the avoidance of wall interactions (De Tommasi et al., 2014) to the optimization of divertor pumping (Calabrò et al., 2015). In a tokamak, the plasma position and shape control task is further complicated by the fact the plasma exhibits a vertical instability, due to the elongated shapes typically pursued (Walker & Humphreys, 2009) (see the elongated cross-section of the ITER plasma reported in Fig. 2).

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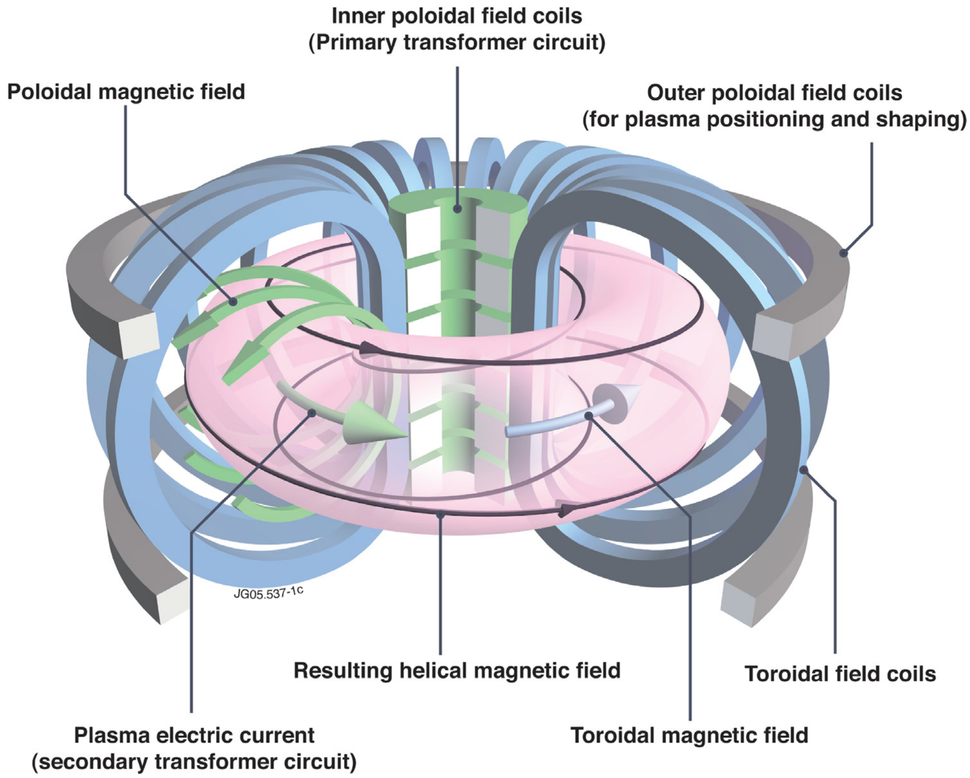


Fig. 1. Simplified scheme of a tokamak fusion device. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Different model-based approaches have been proposed in literature to solve the vertical stabilization problem in a robust fashion, including nonlinear adaptive control (Scibile & Kouvaritakis, 2001), MPC (Gerkšič & De Tommasi, 2013) and multi-objective optimization techniques (De Tommasi, Mele and Pironti, 2017). In many cases the adopted control approach has been tailored taking into account the features of the specific experimental device: see Sartori, De Tommasi, and Piccolo (2006) for the vertical stabilization of the JET tokamak, Schuster, Walker, Humphreys, and Krstić (2005) for the DIII-D system, and Ambrosino, Ariola, De Tommasi, and Pironti (2011a) for a model-based ITER VS system that has been also tested on the EAST tokamak (Albanese et al., 2017; De Tommasi, Mele, Luo, Pironti and Xiao, 2017). Such a *customization* is needed since the performance of any existing VS system strongly depends on the *growth rate* γ of the instability, usually defined as the unstable eigenvalue of the linearized plasma response model obtained around the considered configuration. The eigenvector associated to γ describes the behaviour of the plasma and of the currents in the passive structures along the unstable direction. A possible VS control approach could be to adapt the control gains as function of γ. However, the estimation of the unstable eigenvalue is based on the real-time reconstruction of the plasma equilibrium (Bao et al., 2020), which is still a computationally demanding task, if compared with the time scale the VS system should react in. One possibility to achieve robust performance in present tokamaks is to adapt the VS parameters according to an empirical relationship between γ and some measurements. This is done for example at JET (Sartori et al., 2006), where the adaption mechanism is based on the switching frequency of the power amplifier that feeds the control circuit. An alternative approach is to design the VS parameters for an envelope of possible plasma models, taking into account different shapes, and different internal current distributions. This approach has been followed in Ambrosino, Ariola, De Tommasi, and Pironti (2011b), where a set of linear models is considered for the design of a robust set of gains for the ITER VS. A similar approach has been adopted for the EAST tokamak in De Tommasi, Mele and Pironti (2017), by exploiting multi objective optimization. However, robust stability is usually achieved at the expenses of the performance robustness, which depends on the set of models considered during the design. As a result, very often the design is practically carried out by means of a trial-and-error procedure, aimed at finding a good trade-off between robust stability and robust performance. Similar considerations hold when nonlinear control approaches are considered, as in the case of JET (Scibile & Kouvaritakis, 2001) and TCV (Cruz et al., 2015).

In view of developing VS control strategies that guarantee the required level of performance, without heavily relying on the knowledge of a plant model, a possibility is to resort to model-free approaches. To this aim, in De Tommasi, Dubbioso, Mele, and Pironti (2021) we introduced a VS system that exploits the ES-based approach originally proposed in Scheinker and Krstić (2017). In particular, in their work, Scheinker and Kristić have shown that it is possible to stabilize *on average* an unstable plant by minimizing a properly chosen candidate Lyapunov function of the plant state with a suitable choice of the control action. In this paper we extend the preliminary results presented in De Tommasi et al. (2021) along different directions. First of all we consider the case where a nonlinear switching power supply is present in the control system, while the linear model case corresponding to the envisaged thyristor bridge converter was considred in De Tommasi et al. (2021). Moreover, by means of extensive linear and nonlinear simulations, we show that it is possible to achieve the required robustness during the whole flat-top of an ITER discharge by exploiting the *model agnosticity* of the ES. Indeed, it is demonstrated how a limited knowledge of the plasma behaviour (i.e. a single and reduced order linear model) is sufficient to design a VS algorithm that works for the entire flat-top, despite the variation of plasma shape and current density

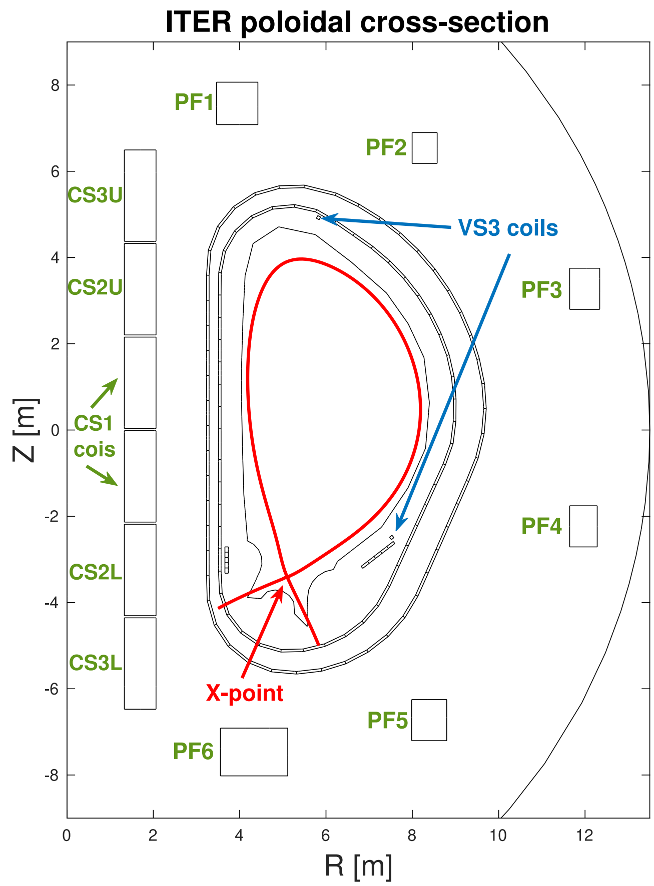


Fig. 2. Poloidal cross-section of an elongated ITER tokamak plasma. The red curve shows the boundary of an elongated plasma. The coils of the superconductive PF circuits are labelled in green, while the blue labelled in-vessel coils are the ones that form the *VS*3 circuit, which is the actuator for the ITER Vertical Stabilization system. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

distribution. As it will be discussed in what follows, this unique linear model is required to design a reduced order Kalman filter which estimates the plasma motion along the unstable mode.

As already mentioned, the validation of the proposed ES-based vertical stabilization approach and the assessment of its robustness are carried out by both linear and nonlinear simulations. By means of the linear simulations we prove that the proposed approach robustly stabilizes a broad family of different plasma models, although the embedded Kalman filter is always the same. On the other hand, nonlinear simulations that involve the solution of a free boundary evolutionary problem are used to show the robustness of the ES-based VS throughout the overall ITER discharge.

It is worth to remark that the considered simulation setup includes the complete ITER magnetic control system, i.e. it also takes into account the interaction of the proposed ES-based VS with the plasma current and shape controllers. Moreover, the ES-based approach does not necessarily require the use of a linear power amplifier for the PF circuit used for VS. Therefore, in the presented simulations we consider a model of a switching power supply, which provides a faster time response with respect to linear amplifiers.

The proposed approach can be regarded as an *almost* model-free one. Although there is an interest in developing model-free plasma control techniques, to the best of the authors knowledge data-driven vertical stabilization approaches are still missing in the literature, even though this kind of techniques have been proposed for tracking problems, such as the control of plasma internal profiles (Shi et al., 2017; Wakatsuki et al., 2019).

The rest of the paper is structured as follows: Section 2 introduces the modelling and simulation environment used to perform both the validation and the performance assessment of the proposed stabilization approach. The magnetic control problem in tokamak fusion devices and a possible architecture for the ITER plasma magnetic control system are presented in Section 3, and the vertical stabilization problem, with a particular focus on ITER, is discussed in Section 3.1. Then, the proposed ES-based control algorithm to solve such stabilization problem is described in Section 4, while the simulation results are discussed in Section 5. In particular, Section 5.1 describes the simulations performed over a family of different linearized models, while in Section 5.2 a set of nonlinear simulations is described, with the aim of validating the proposed approach. Eventually, some conclusive remarks are given.

2. Nonlinear modelling of plasma/circuit dynamics

Mathematical modelling of tokamak plasmas for magnetic control validation is based on the so-called Grad–Shafranov partial differentialequation (GS-PDE, Shafranov (1966)). Because of the low plasma mass density, inertial effects can be neglected and, as a consequence, the plasma momentum equilibrium equation becomes $J \times B = \nabla p$. This equation can be rewritten, in axial-symmetric geometry with cylindrical coordinates (r, ϕ, z), as:

$$\begin{aligned} &\Delta^* \psi = -f\frac{df}{d\psi} - \mu_0 r^2 \frac{dp}{d\psi} && \text{in the plasma region} \\ &\Delta^* \psi = -\mu_0 r j_{ext}(r, z, t) && \text{in the conductors} \\ &\Delta^* \psi = 0 && \text{elsewhere} \end{aligned} \tag{1}$$

with boundary conditions:

$$\psi(r, z, t) = \psi_0(r, z),\ \psi(0, z, t) = 0,\ \lim_{r^2+z^2\to\infty} \psi(r, z, t) = 0, \forall t \tag{2}$$

where $\psi = \psi(r, z)$ is the poloidal flux per radian, μ_0 is the vacuum magnetic permeability, j_{ext} is the toroidal current density in the external conductors (both control coils and passive structures), $p = p(\psi)$ is the kinetic pressure profile, an $f = f(\psi)$ is the poloidal current function profile, and the Δ^* operator is defined as:

$$\Delta^* = r\frac{\partial}{\partial r}\left(\frac{1}{\mu_r r}\frac{\partial \psi}{\partial r}\right) + \frac{\partial}{\partial z}\left(\frac{1}{\mu_r}\frac{\partial \psi}{\partial z}\right). \tag{3}$$

Solutions of the GS-PDEs can be numerically found by means of numerical integration techniques such as Finite Element Methods (FEMs), provided that the plasma boundary can be determined, the toroidal current densities in the PF coils and the total plasma current are known, functions $p(\psi)$ and $f(\psi)$ are defined. j_{ext} can be expressed as a linear combination of the circuit currents, the time evolution of which is given by a circuit equation in the form:

$$\dot{\Psi} + RI = V \tag{4}$$

Hence, it can be shown that:

$$j_{ext} = -\frac{\sigma}{r}\dot{\psi} + \frac{\sigma}{2\pi r}u \tag{5}$$

where u is the voltage applied to the coils (zero for the passive structure) and σ is the electric conductivity. Eq. (5) must be integrated over the conductor regions.

Once the ψ map evolution is known, it is possible to compute other variables of interest for control as plasma current I_p, plasma position and plasma shape. Shape is then described by means of plasma-wall distances at given points (plasma-wall gaps) (Beghi & Cenedese, 2005) which are usually controlled.

In practice, the GS-PDE is solved by using numerical solvers, and in the present work the CREATE-NL+ nonlinear magnetic equilibrium code (Albanese, Ambrosino, & Mattei, 2015) is used. This code is exploited in Section 5 to assess the performance of the proposed ES-based VS system for the ITER case.

The difficulty of using nonlinear FEM models for control design purposes makes a linearization procedure of the plasma response necessary. Following the procedure described in Albanese and Villone (1998), we can finally obtain a linear plasma-circuit dynamics in the form:

$$\begin{aligned} L\delta\dot{I} + R\delta I &= \delta V + L_E \delta\dot{w} \\ \delta y &= C\delta I + F\delta w \end{aligned} \tag{6}$$

where δV is a vector containing the voltages applied to the circuits (zero for the passive structures) and $\delta I = [\delta I_A^T, \delta I_E^T, \delta I_p]^T$ is the vector of PF, passive, and plasma currents respectively; R is the circuit resistance matrix and L is the matrix of self and mutual inductances between the plasma, the coils and the equivalent circuits modelling the passive structures; δw is a vector of parameters describing plasma current density profile, typically assumed as external disturbances (the so called poloidal beta β_p and internal inductance l_i are often chosen); δy is a vector of outputs and C and F are suitable output matrices. All the quantities in which the symbol δ appears are intended to be variations with respect to the equilibrium value (i.e. the nominal conditions around which the linearization is made).

Assuming:

$$x = I - L^{-1} L_E w$$

Eq. (6) can be rewritten in the state space form as:

$$\begin{aligned} \delta\dot{x} &= A\delta x + B\,\mathrm{sat}(\delta V) + E\delta w \\ \delta y &= C\delta x + F\delta w \end{aligned} \tag{7}$$

with clear meaning of symbols and matrices. The limitation of voltage has been accounted for by introducing a saturation function, namely $\mathrm{sat}(\cdot)$.

3. The plasma magnetic control architecture

In this section the plasma magnetic control problem in a tokamak is briefly discussed, with particular attention to the vertical stabilization problem.

The simplified block diagram of a possible magnetic control architecture is reported in Fig. 3. This architecture, other than being broadly adopted in many operating tokamaks, such as JET (Sartori et al., 2006) and EAST (Albanese et al., 2017), is also the one currently considered for ITER (Ambrosino et al., 2015; Cinque et al., 2020).

As already mentioned in Section 1, the confinement of the hot plasma in a tokamak device is achieved by means of magnetic fields through the pulse phases defining the so-called plasma scenario. In particular, the magnetic field produced by the PF coils is needed from the start of the discharge to achieve the conditions for plasma formation inside the vacuum chamber (the so called breakdown and burn-through phases (Jackson, Humphreys, Hyatt, & Leuer, 2011)). Soon after plasma formation, the currents flowing in the PF coils need to be controlled in order to increase the plasma current during the ramp-up phase, to keep it almost constant during the so-called flat-top, and then to ramp it down during the final phase of the discharge. In addition to the control of the plasma current, also the plasma boundary and position need to be controlled to achieve the desired experimental objectives. Moreover, in the case of vertically elongated plasmas, as in the case of ITER, the active control of the current in some of the PF coils is mandatory in order to generate the radial field needed to vertically stabilize the plasma column (Lazarus, Lister, & Neilson, 1990; Walker & Humphreys, 2009).

The main components of the plasma magnetic control system shown in Fig. 3 are briefly described hereafter.[1]

- The **PF Current (PFC) Decoupling Controller**, this block acts as the inner control loop of a nested architecture that includes also the plasma current and shape controllers. By generating the required voltages to be applied to the superconductive coils, this block tracks the PF current references, which are a sum of the *scenario* (i.e., the nominal) currents and the corrections requested by the outer loops to track the desired plasma shape and current;
- The **Plasma Current Controller**, which tracks the plasma current reference by sending the correspondent requests to the PFC Decoupling Controller;
- The **Plasma Shape Controller**, which controls the shape of the last closed flux surface within the vacuum chamber by tracking a set of plasma shape descriptors; this block also generates requests for the PFC Decoupling Controller.
- The **Vertical Stabilization** (VS) system, which is in charge of vertically stabilizing the plasma column. More details about this block are given in the next section.

3.1. The vertical stabilization problem

High performance plasmas have a diverted shape (i.e., with an *active* X-point in the vacuum chamber) with an elongated poloidal cross-section, as the one shown in Fig. 2. Elongated plasmas provide considerable advantages on energy confinement and achievable pressures. The price to be paid to improve the fusion performance is that such elongated plasmas are vertically unstable (a simple description of such instability can be found in De Tommasi (2019)).

The plasma vertical instability reveals itself in the linearized model of the plasma behaviour (7) by the presence of an unstable eigenvalue. Thanks to the presence of the conducting structures that surround the plasma, the instability characteristic time is brought to a scale that can be controllable via active stabilization circuits.

It follows that the Vertical Stabilization block in Fig. 3 is an essential component of the magnetic control algorithm to run tokamak discharges with an elongated plasma. The stability must be guaranteed in the presence of uncertainties and time varying behaviour of the plasma along the scenario, and good performance of the overall plasma magnetic control system should be guaranteed in the presence of disturbances, such as Edge Localized Modes (ELMs) or Minor Disruptions (MDs), and other fast disturbances modelled as Vertical Displacement Events (VDEs); for more details refer to Section 5. In practically all the existing tokamaks, the VS system drives a combination of currents in a set of dedicated PF circuits that produces a mainly radial magnetic field which is needed to apply the vertical force used to stop the plasma column. In ITER such dedicated circuit is the so called *VS3* circuit (see again Fig. 2), which is made by one pair of coils fed in anti-series.[2]

VS control algorithms with a simple structure and few control parameters are usually preferred on existing machines (Albanese

[1] For more details on the control algorithms implemented by the various blocks shown in Fig. 3, the interested reader can refer to Ariola and Pironti (2016) or De Tommasi (2019).

[2] Two coils are said to be connected in anti-series if they are connected together with the winding, and hence the flowing current, in opposite direction.

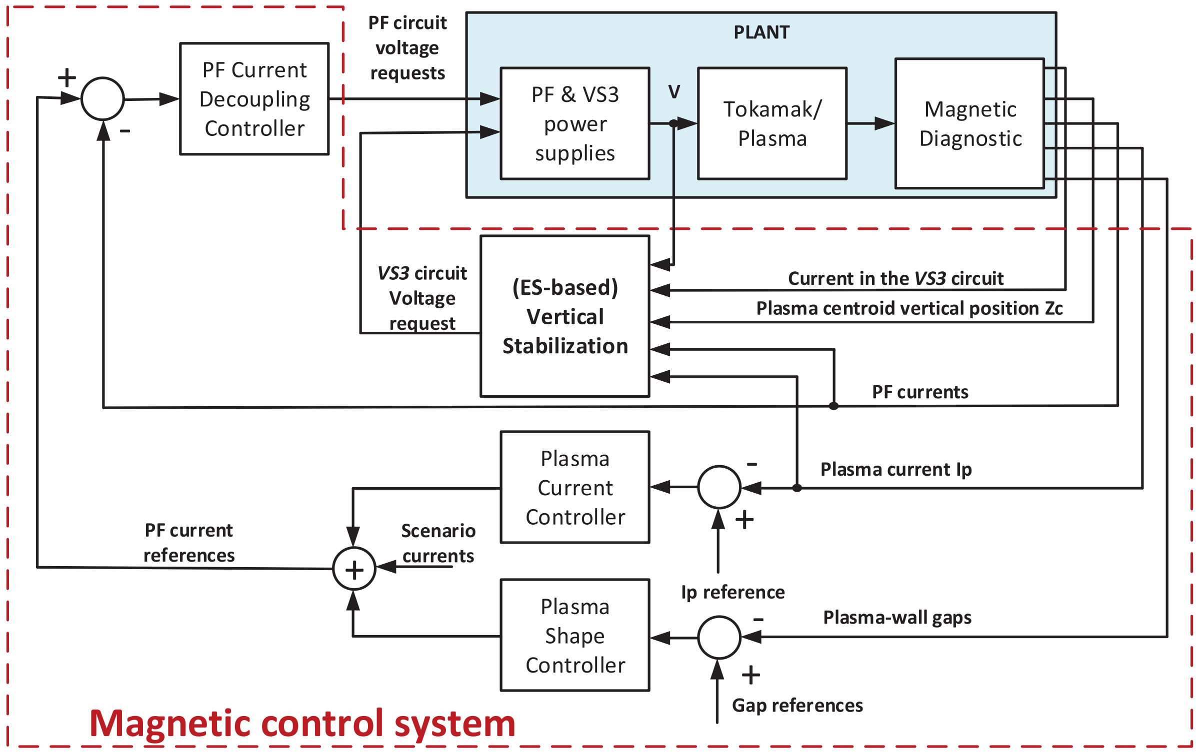


Fig. 3. Block diagram of a typical ITER-like architecture for plasma magnetic control in tokamaks (Ambrosino et al., 2015; Cinque et al., 2020). The ES-based VS algorithm proposed in Section 4 is meant to be deployed in the *Vertical Stabilization* block reported in this diagram.

et al., 2017). Indeed, a simple structure enables the deployment of effective adaptive algorithms, aimed at robust operations under various scenarios (Neto et al., 2012). However, such adaptive algorithms are not always straightforward to design, either because they require reliable models, which are not necessarily available, or because their tuning requires a considerable effort in terms of time, as well as a considerable experience on the specific machine. Therefore, in the next section, we propose an ES-based VS system that aims at achieving the requested robustness without the need of a detailed model, thanks to the model-agnostic nature of the ES algorithm.

4. ES-based plasma vertical stabilization

The ES-based architecture for the VS of tokamak plasmas proposed in this work is based on the approach described in Scheinker and Krstić (2017) to stabilize an unknown, unstable system. However, in order to apply it to solve the VS problem, an estimation of the state motion along the unstable mode is needed; one possibility to obtain such estimate is to use an observer such as a Kalman filter.

The ES method presented in Scheinker and Krstić (2017) aims at achieving stabilization via minimization of a candidate Lyapunov function of the unstable system, assuming that the state can be measured or estimated. In particular, it has been shown that given the nonlinear system affine in control

$$\dot{x}(t) = f(x,t) + g(x,t)u(t),$$

it is possible to employ a nonlinear time-varying control law in the form

$$u(t) = \alpha\sqrt{\omega}cos(\omega t) - k\sqrt{\omega}sin(\omega t)V(x)\,, \tag{8}$$

where $V(x)$ is a Lyapunov function, used to stabilize the associated Lie-bracket average system

$$\dot{\bar{x}}(t) = f(\bar{x},t) - \frac{k\alpha}{2}\,g(\bar{x},t)g^T(\bar{x},t)\left(\frac{\delta V(\bar{x})}{\delta \bar{x}}\right)^T, \tag{9}$$

In fact, from Eq. (9) it can be seen how a choice of a sufficiently high positive gain $k\alpha$ makes the gradient term dominant and the average system asymptotically stabilized.

Moreover, it can be shown, via averaging arguments, that the trajectories of system (4) under the control input (8) can be kept arbitrarily close to those of (9), provided that the frequency ω is chosen high enough. This guarantees that all the trajectories of the original system are confined to a neighbourhood of the averaged ones, making the system semi-globally practically stabilized (more details can be found in Scheinker and Krstić (2017), Moreau and Aeyels (2000) and Teel, Peuteman, and Aeyels (1999)).

Although the stabilization via ES does not require the knowledge of the system, it requires that the value of the function $V(\cdot)$ is known, which means that system's state must be accessible. For a tokamak plasma, like the ITER one, only a subset of the state of the associated linearized system (7) can be measured or readly estimated in real-time with a static combination of measurements, which in this case consists in the PF currents I_A and the plasma current I_p, while the eddy currents would require a dynamic estimator. On the other hand, as a matter of fact, is not possible to define a candidate Lyapunov function disregarding the eddy currents which play a fundamental role in the dynamics of the vertical instability.

Therefore, in this work, we propose to use a candidate Lyapunov function based only on the estimation of the state dynamics along the unstable mode of the linearized model (7). This estimation is achieved by means of a Kalman filter. Although such a filter requires the knowledge of a model, in Section 5 it is shown that the proposed architecture can cope with relevant model uncertainties, since it anyhow exploits the *model-agnostic* nature of the ES algorithm (8).

A scheme of the ES-based VS architecture implemented for ITER is shown in Fig. 4. The control output of the stabilization system u_1, is the voltage request to the ITER VS circuit $VS3$. The other input to the plant, u_2, is a vector containing the voltages

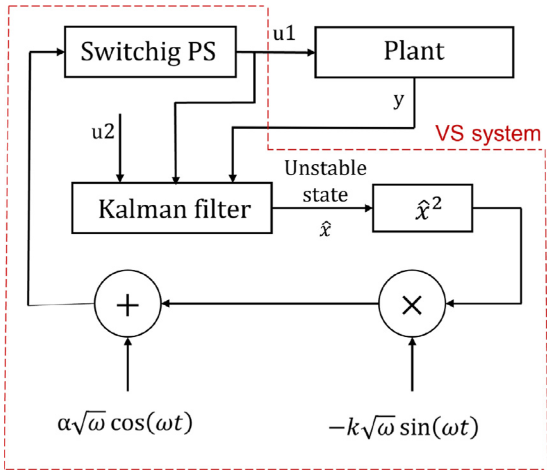


Fig. 4. The proposed VS system based on the ES stabilization algorithm and the switching power supply.

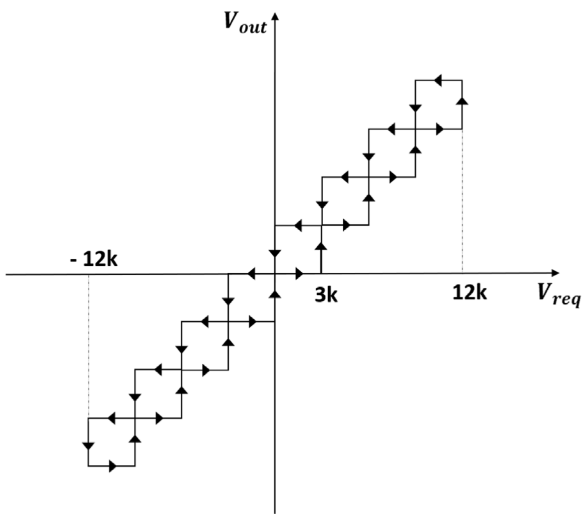


Fig. 5. Characteristic of the switching power supply.

applied to the superconductive PF circuits by the PF Current Decoupling Controller (see Fig. 3). For the proposed application, the considered plant output vector y is defined as

$$y = \left(\delta I_A^T \quad \delta I_p \quad \delta Z_c\right)^T,$$

i.e. it contains the variations, with respect to the equilibrium value, of the currents in the active circuits δI_A (both the superconductive PF and the $VS3$ circuits), of the plasma current δI_p and of the vertical position of the plasma centroid δZ_c. It is worth to observe that, in a real tokamak, the active currents I_A can be directly measured, while the plasma current and the centroid vertical position are usually obtained as a linear combination of the available magnetic field measurements. On the other hand, for what concerns the plant inputs, the equilibrium voltages are all equal to zero, therefore the variations of u_1 and u_2 coincide with their actual values.[3]

The Kalman filter receives as input u_1, u_2 and y and provides an estimation of the dynamics along the unstable mode $\hat{x}$. It has been designed assuming high confidence in the measurements,

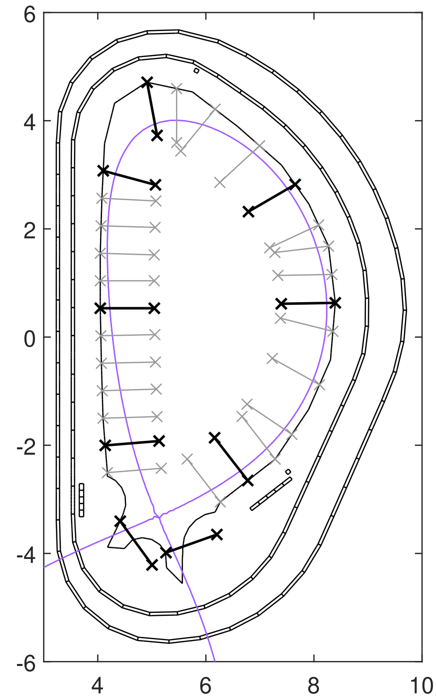


Fig. 6. Gaps used for the shape control. The gaps shown in black are the ones whose behaviour is reported in Figs. 10–14.

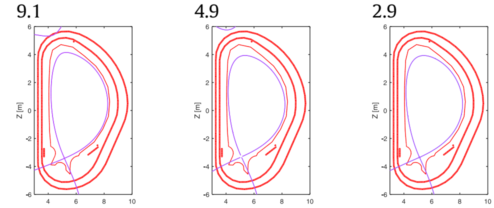
Table 1
Main parameters of the fast switching power supply.

Parameters	ERFA
Max output voltage	±12 kV dc
Max output current	±5 kA dc
Max output voltage step	±3 kV
Time for full ± voltage excursion	≤ 100 μs
Max switching frequency	1 kHz

Table 2
Control parameters for the proposed model-free VS system based on the ES control law (8).

k	α	ω
$2.7 \cdot 10^{-3}$	1	$250 \cdot 2\pi$ rad/s

Table 3
ITER equilibria considered for the nonlinear simulations, with the corresponding plasma boundary.

Configuration	Eq #1	Eq #2	Eq #3
I_p [MA]	14.7	15	15
β_p	0.08	0.66	0.81
l_i	0.92	0.88	0.71
γ [s^{-1}]	9.1	4.9	2.9
			

which is reflected in the choice of almost negligible covariance matrices. The estimation of $\hat{x}$ returned by the Kalman filter is used to compute the candidate Lyapunov function $V(\hat{x}) = \hat{x}^2$ to be minimized by the ES control algorithm.

[3] In the first approximation, the plasma is assumed to be superconductive, so there is no need to ramp the PF currents in order to compensate for the ohmic drop.

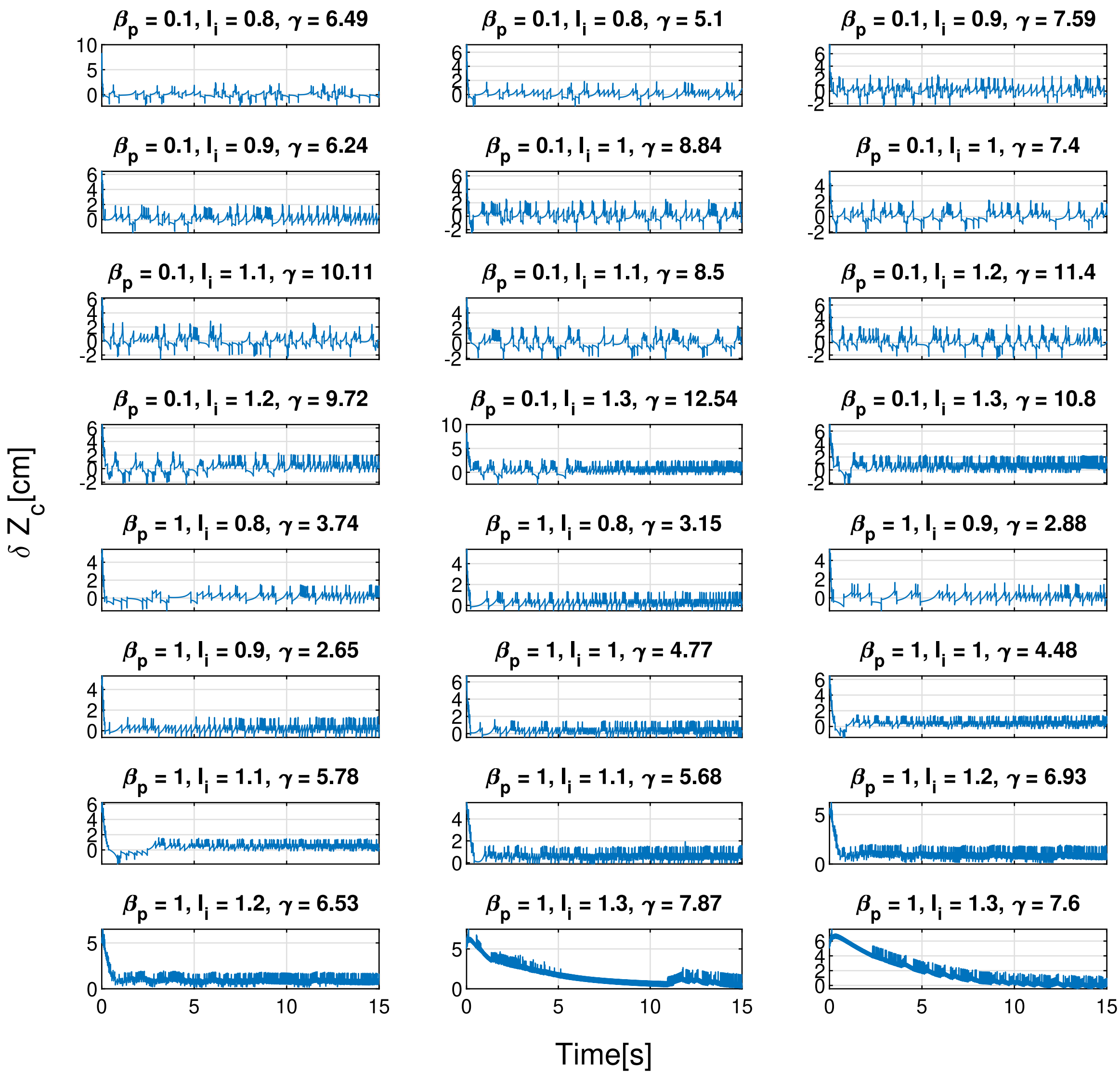


Fig. 7. Response to a VDE of 5 cm for the considered family of different plasma models in terms of the displacement from the equilibrium value of the plasma vertical position Z_c.

The tuning of the control gains k and α in (8) can be carried out with a trial and error procedure by means of numerical simulations. However, a first guess for the product $k\alpha$ has been obtained by considering the first order reduced model that links the voltage applied to the vertical stabilization circuit to the unstable state (i.e. the unstable dynamics alone). Indeed, when such first order reduced model is considered, from (9) it readily follows that the closed loop average system is equal to $\dot{\bar{x}} = \left(a_{red} - k\alpha\, b_{red} b_{red}^T\right)\bar{x}$; therefore, if $a_{red} > 0$ is the unstable eigenvalue of the reduced system, the corresponding average system is stable if the product $k\alpha$ is sufficiently high.

In order for the averaging arguments that lead to (9) to be valid, the frequency ω must be chosen "high enough"; for this reason, as it is commonly found in averaging analyses, the resulting system exhibits an intrinsic time-scale separation. In fact, in De Tommasi et al. (2021) a first attempt to apply this technique to the tokamak vertical stabilization problem was proposed, where a linear power amplifier was employed. As a result, the switching frequency ω was limited due to the bandwidth of the power supply. Conversely, in this paper we assess the impact of using a switching power supply similar to the one used for the JET VS system, which is based on the integrated gate commuted thyristors (Toigo et al., 2007). Indeed, the availability of a faster actuator enables the choice of a higher switching frequency ω for the mixing and dithering terms in (8), leading to an improvement of the performance with respect to what preliminary presented in De Tommasi et al. (2021).

The characteristic of this kind of power supply, which exhibits a multi-level hysteresis, is reported in Fig. 5. As for the parameters of the power supply, we considered the same of the one currently used at JET, which has a maximum voltage of 12 kV, with steps of 3 kV (see Table 1).

The power supply considered in this work has been simulated taking into account the maximum output voltage and voltage steps reported in Table 1, with an internal delay of 200 μs.

5. Simulations of the proposed control algorithm

The proposed ES-based VS has been tested by both linear and nonlinear simulations, with the aim of proving the validity of the approach and to assess its robustness. In particular, since there is no *a priori* guarantee that the estimate of the unstable dynamic provided by the Kalman filter is accurate enough for the proposed VS to stabilize the plant, a robustness assessment is carried

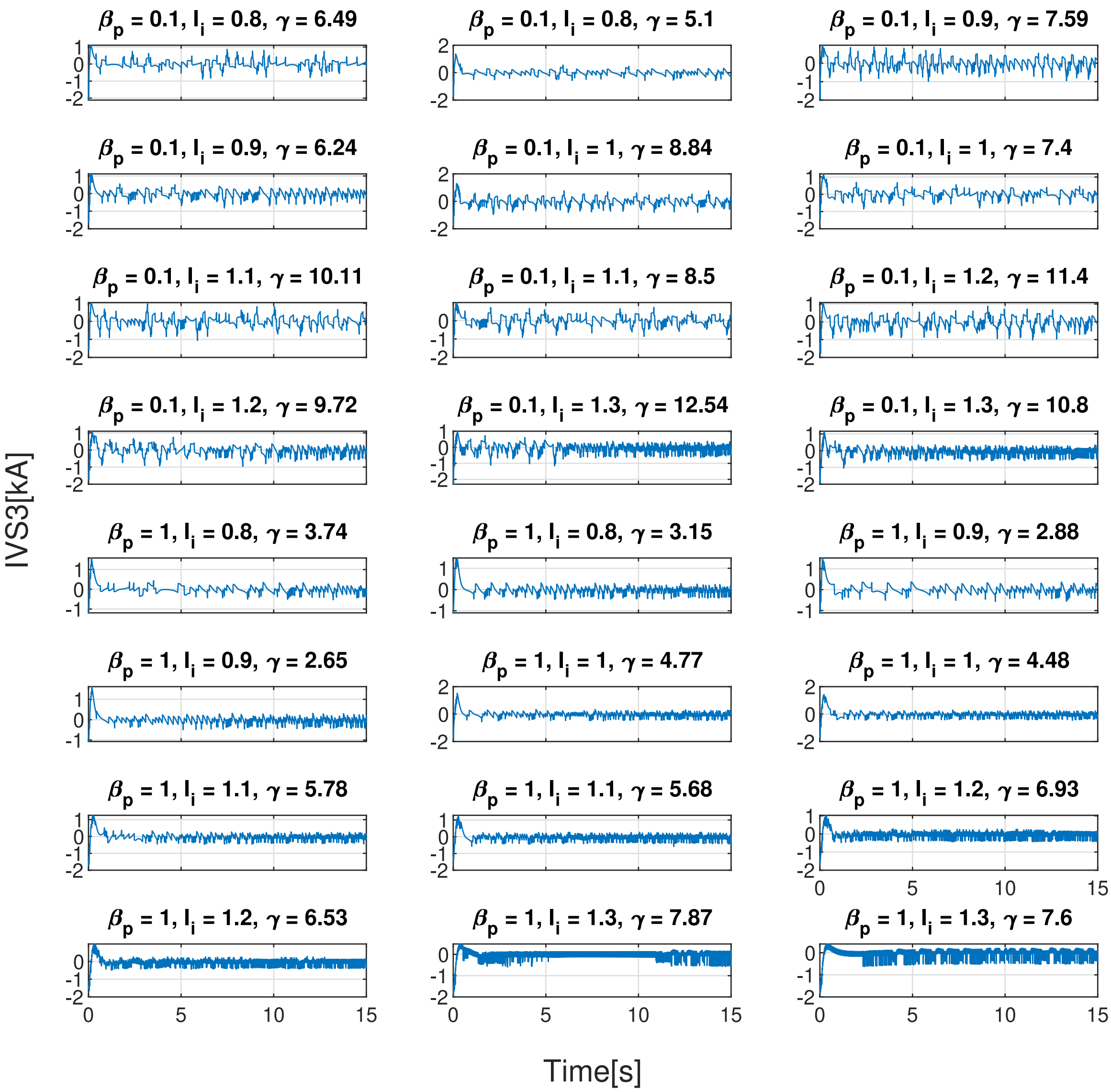


Fig. 8. Response to a VDE of 5 cm for the considered family of different plasma. The time behaviour of the currents in the VS system coils *IVS*3 have been reported.

out *a posteriori* by means of numerical simulations which cover a significant variety of different plasma configurations/parameters.

As already pointed out in Section 1, the control architecture exploited for the simulations includes the overall ITER magnetic control system, whose scheme is reported in Fig. 3; therefore the interaction of the proposed ES-based VS with the plasma current and shape control systems is also taken into account. In particular, the shape control algorithm adopted is the so-called eXtreme Shape Controller, which in this case controls 29 plasma-wall distances (the *gaps* shown in Fig. 14), with a settling time of about 10 s. In order to control a number of plasma shape descriptors, i.e. the 29 gaps, which is greater than the number of available actuators (i.e., the 11 currents in the superconductive coils), the XSC design is based on a Singular Value Decomposition of the static relationship between the control inputs and outputs. In this way, it is possible to minimize in least mean square sense the control error at steady-state. For more details about the XSC the interested reader can refer to Albanese et al. (2005) and Ariola and Pironti (2005).

Moreover, the simulation have been performed considering a set of operational scenarios for the ITER tokamak. The considered scenarios refer to the counteraction of relevant disturbances that can occur during plasma operation. In particular, the following cases were considered:

- the rejection of a VDE of 5 cm;
- the response to a MD.

A VDE is an instantaneous motion of the system's state along the unstable mode, scaled so as to produce a prescribed vertical displacement of the plasma centroid. Although, the plasma is always vertically controlled, a VDE is a standard benchmark to assess VS performance (Ambrosino et al., 2011a), since it models various type of disturbances, such as unforeseen delays in the control loop and wrong control action due to measurement noise, when plasma velocity is almost zero. After a VDE, the VS system must be able to bring back the centroid to the reference position (i.e. zero displacement with respect to the equilibrium value)

A MD represents instead a lost of a fraction of the plasma thermal energy, due to the uncontrolled growth of some plasma instability. For this application, a MD can be modelled as an instantaneous drop of 0.1, from the nominal values, of the disturbance parameters β_p and l_i (see also Corona et al. (2019, Sec. 3)).

For all the considered scenarios, the same configuration of the VS system has been used. This means that, in all simulations,

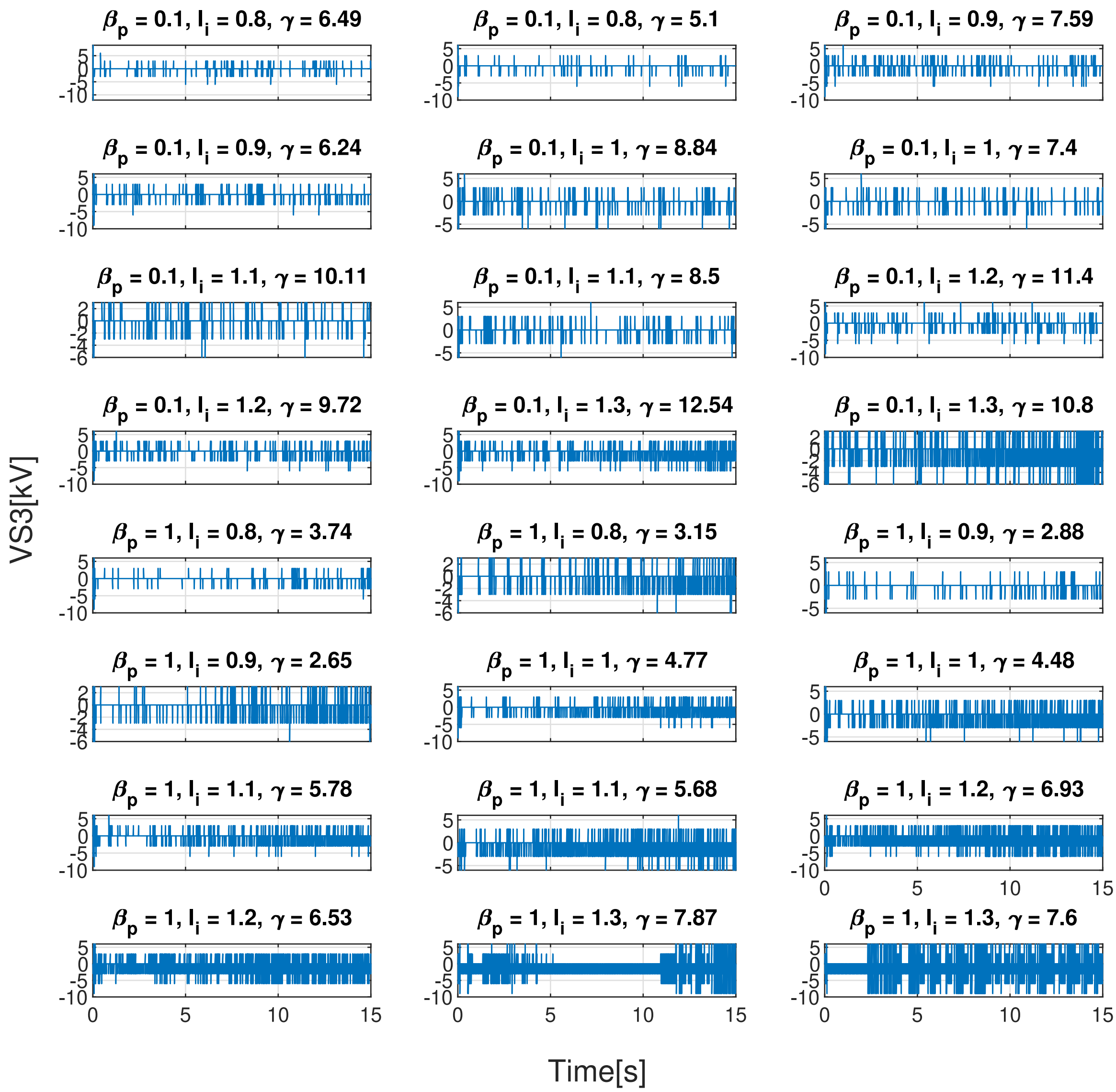


Fig. 9. Response to a VDE of 5 cm for the considered family of different plasma model. The figure reports the voltages applied to VS system, $VS3$.

the same Kalman filter was taken into account, as well as the same ES parameters. The value of the ES control parameter in the control law (8) is reported in Table 2, while the parameters of the switching power supply are the shown in Table 1.

5.1. Linear simulation validation

The linear simulations have been carried out with the VS system architecture presented in Section 4. The aim was to prove that the designed ES-based approach can stabilize a broad family of different plasma models, although the embedded Kalman filter is always the same.

The considered family of plasma models consists of 24 different plasma equilibria, all at a plasma current of 15 MA, generated so as to cover the interval

$$(l_i, \beta_p) \in [0.8,\ 1.3] \times [0.1,\ 1],$$

with two different plasma shapes, characterized by two slightly different elongations $\kappa = 1.81, 1.76$, respectively.[4]

The unique Kalman filter adopted for linear simulations was obtained considering a reduced linearized model, of order 25, for the equilibrium characterized by $l_i = 1.3$, $\beta_p = 1$ and a growth rate $\gamma = 7.6\ \mathrm{s}^{-1}$. The operational scenario considered for the linear simulations is a rejection of a VDE of 5 cm.

The results of the simulations are shown in Figs. 7–10, where the displacement from the equilibrium of the plasma centroid position δZ_c, the current and voltage in the VS coils, $IVS3$ and $VS3$ respectively, and the value of the main gaps, chosen according to Fig. 6, are reported for the family of different plasma models considered.

The results show that, thanks to the model-agnostic nature of the ES algorithm, the proposed architecture is capable of dealing with relevant model uncertainties. Indeed, all the considered plasma configurations can be stabilized with the use of a single Kalman filter designed to estimate a simplified, reduced order dynamics (as discussed in Section 4). This indicates that the proposed VS system can guarantee a satisfactory degree of robustness.

As it can be seen in Figs. 7–10, the worst-case maximum plasma vertical displacement is about 10 cm, while in most of the cases the maximum δZ_c is close to the initial VDE of 5 cm. Furthermore, the Z_c variation is rejected very rapidly in most of

[4] The elongation is defined as $\kappa = \frac{b}{a}$, where b is the plasma height and a is the plasma minor radius (Freidberg, 2007, Ch. 5). Plasma growth rate increases with the elongation.

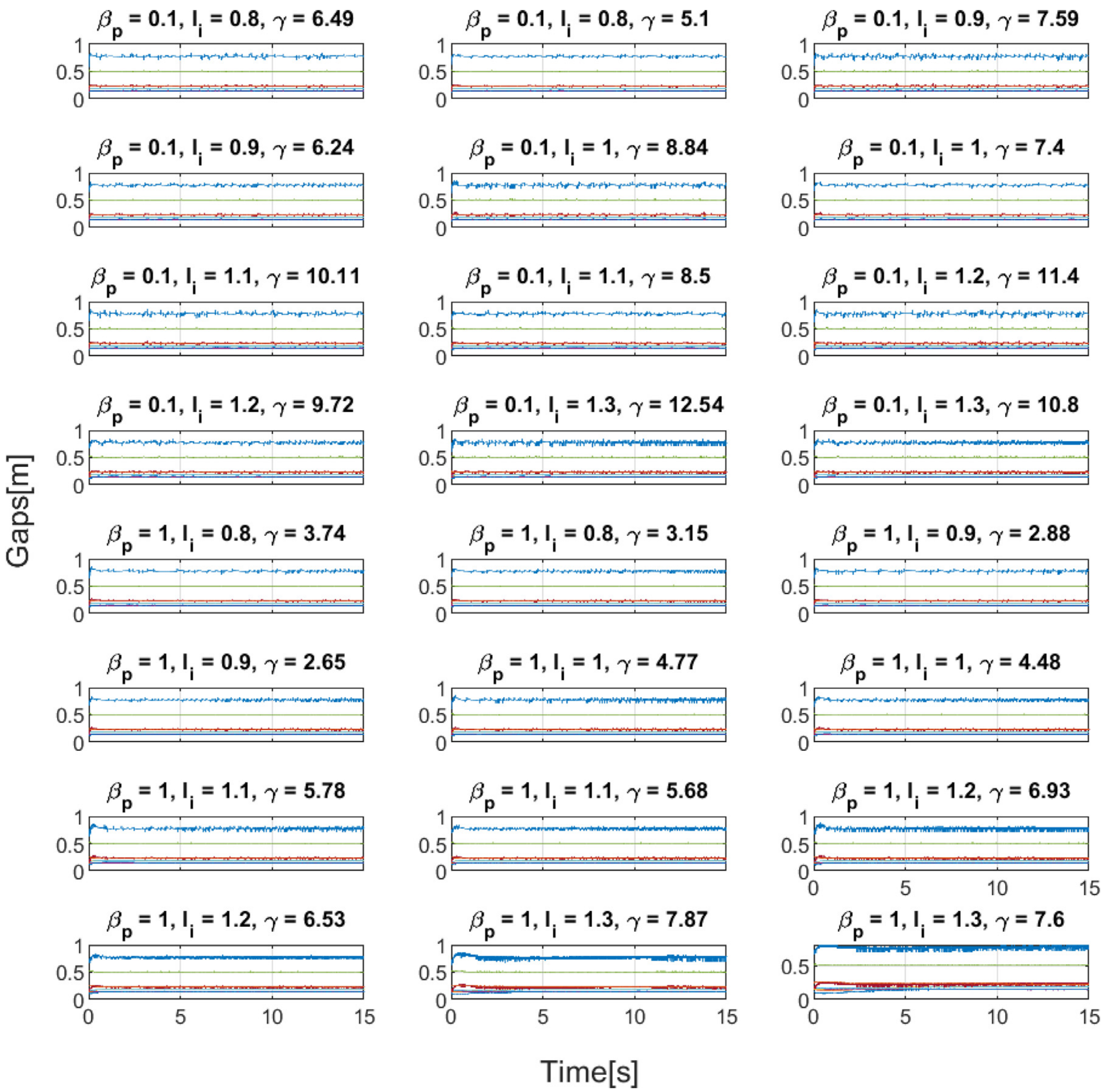


Fig. 10. Response to a VDE of 5 cm for the considered family of different plasma model. The time behaviour of the position of some of the main gaps have been reported.

the considered cases, with the exception of the last two configurations shown in Fig. 7 where the settling times reaches a few seconds. Finally, it is worth to observe that in all the considered cases the maximum in-vessel current is in the order of a few kA.

Plasma current and shape controllers were also included in the simulation scheme, in order to verify that they do not negatively interact with the proposed VS. In fact, it was possible to verify that the plasma current remains practically unchanged (the variations are of the order of a few kA for a plasma current of 15 MA), while the plasma boundary does not touch the vessel during the transient in any of the considered scenarios (Fig. 10 shows that the gaps are always positive, i.e. the plasma boundary never collides with the surrounding walls).

5.2. Nonlinear simulations

Apart from the linear simulation discussed in Section 5.1, nonlinear numerical simulations have been carried out with the CREATE-NL+ free boundary evolutionary code, with the aim of validating the proposed VS control approach on ITER plasma scenarios, in the presence of significant non-linearities and of a more realistic behaviour of the plasma. In the past, the CREATE-NL+ code has been validated against experimental data coming from several tokamaks, including JET (Albanese et al., 2015). For the simulations, the three following starting equilibria have been chosen, whose main parameters are summarized in Table 3:

- an equilibrium at the end of the ramp up phase (*Eq #1* in Table 3).
- an equilibrium at the beginning of the flat-top phase (*Eq #2* in Table 3).
- an equilibrium at the end of the flat-top phase (*Eq #3* in Table 3).

Two different disturbances have been considered:

- VDE of 5 cm;
- MD ($\Delta\beta_p = -0.1$, $\Delta l_i = -0.1$).

It is worth to notice that while the disturbance of a VDE has been applied to every equilibrium, the MD has been considered only for Eq #2 and Eq #3 as the MD is a phenomenon arising at higher values of the plasma energy content represented by β_p.

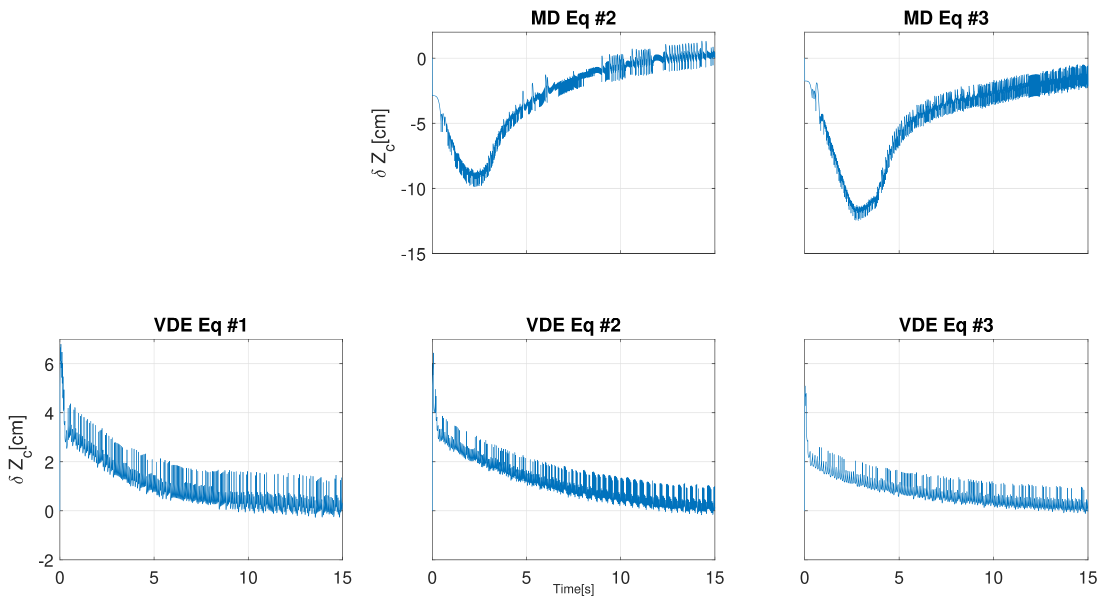


Fig. 11. Nonlinear response to a MD and a VDE for the plasma models in terms of the displacement from the equilibrium values of the plasma vertical position Z_c.

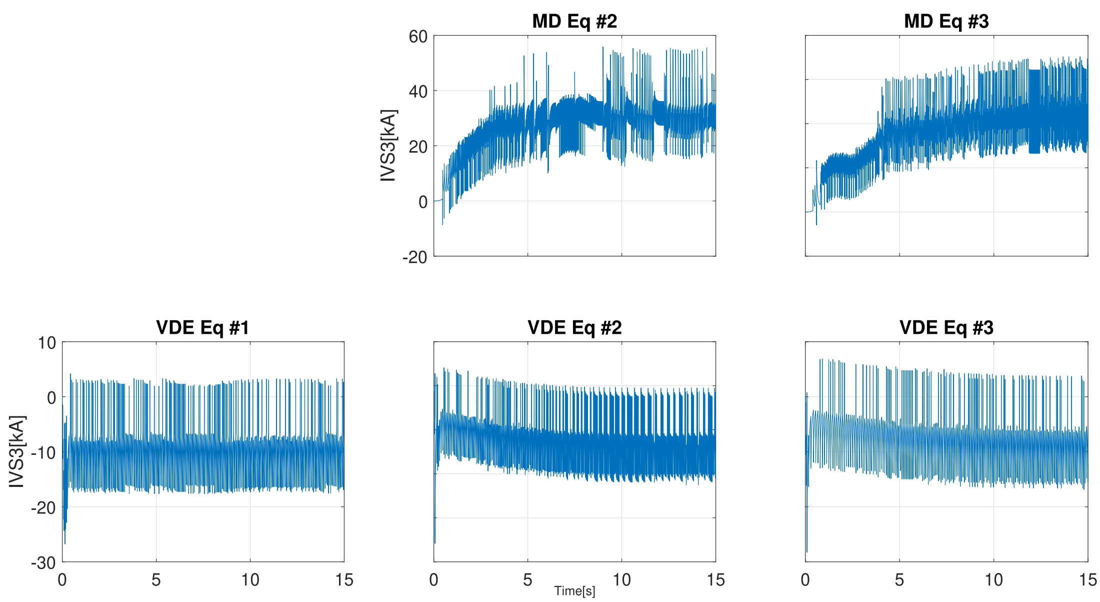


Fig. 12. Nonlinear response to a MD and a VDE for the plasma models in terms of the current in the VS system circuit $IVS3$.

As in the linear case, the simulations have been carried out including all the blocks of the magnetic control architecture shown in Fig. 3, taking into account the interaction of the VS with the plasma current and shape control systems. Moreover, the Kalman filter of the VS scheme (Fig. 4) has been obtained from the reduced linearized model of Eq #2, corresponding to the equilibrium at the beginning of the flat-top phase, while the ES parameters in the control law (8) are the same used in the linear simulations (see Table 2).

In Fig. 11 the displacement of the plasma centroid from the equilibrium position is shown for all the examined cases. It can be noticed that the controllers are able to reject both the considered disturbances starting from the proposed equilibria. The worst case, in terms of δZ_c overshoot, is that of equilibrium #3 in the case of a minor disruption. Moreover, in Fig. 12 the current flowing in the stabilization circuit is shown, while Fig. 13 shows the applied voltage. As expected, the highest current is reached again for *Eq #3* in the case of a minor disruption. Lastly, in Fig. 14 the gaps evolution is presented, showing how the initial shape is restored after the occurrence of the disturbance.

These simulations underline once again the robustness of the proposed model-free architecture.

Conclusions

A model-free approach to tackle the plasma VS problem in tokamak devices has been presented in this paper. The proposed VS architecture consists of a stabilization algorithm based on an ES-like control law and relies on a single, simplified Kalman filter.

The simulation results show that the proposed VS scheme achieves a satisfactory level of robustness during the overall flat-top phase of an ITER discharge and for different plasma configurations. Indeed, the proposed control architecture can practically stabilize the plasma column, by keeping the system state in a bounded set, while counteracting relevant plasma disturbances.

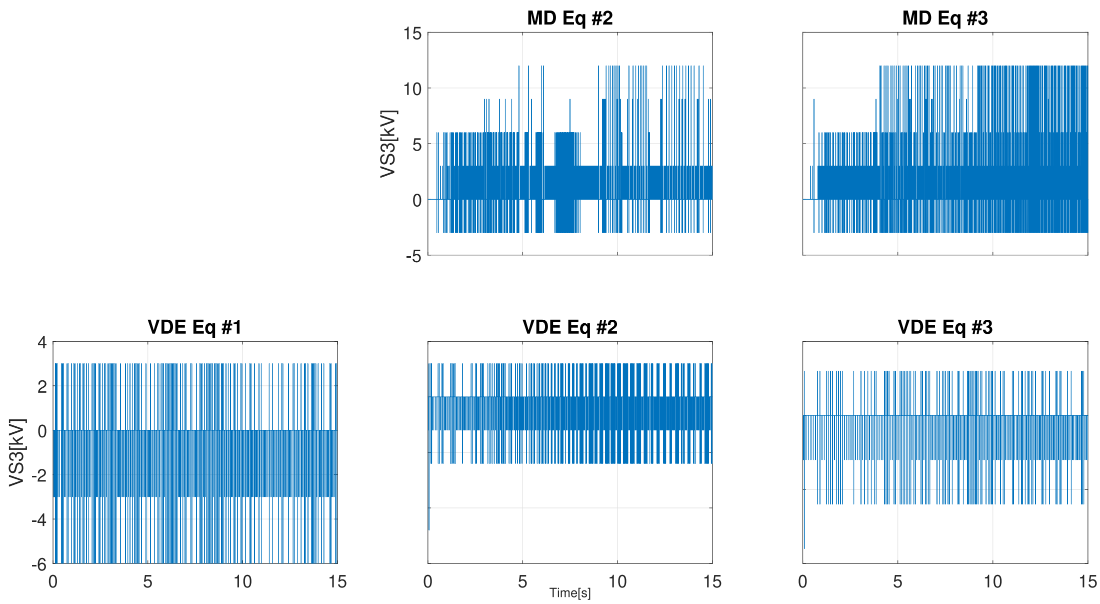


Fig. 13. Nonlinear response to a MD and a VDE for the plasma models in terms of the voltage applied to the VS system circuit.

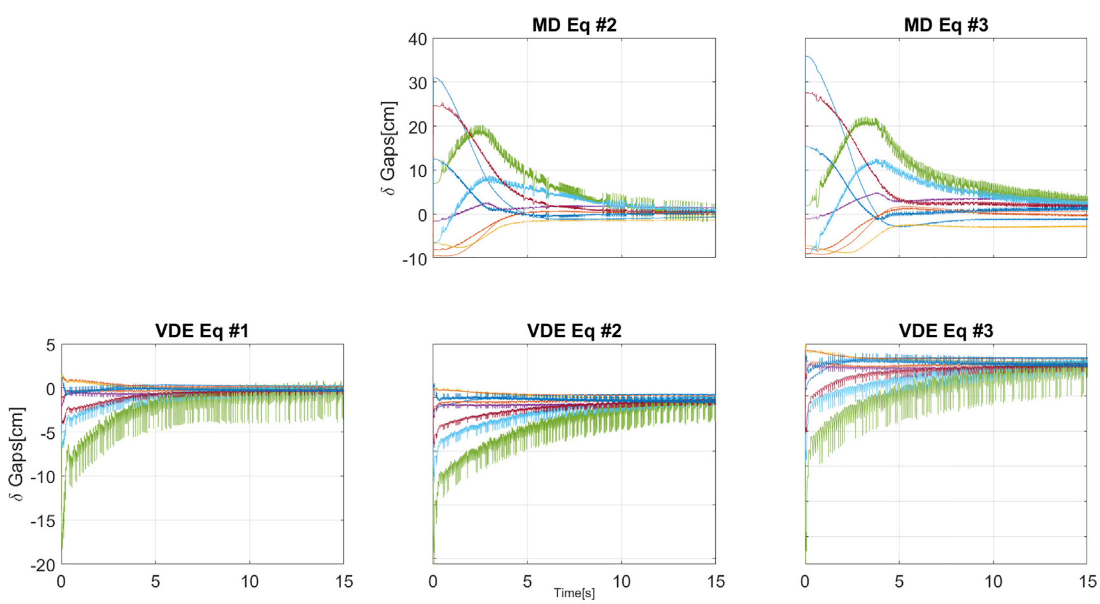


Fig. 14. Nonlinear response to a MD and a VDE for the plasma models in terms of some of the controlled gaps.

Moreover, the *model-agnostic* nature of the ES algorithm allows to cope with large model uncertainties, due to the different plasma equilibria considered. In addition, thanks to the inclusion of the plasma current and shape controllers in the simulation scheme, it was possible to verify that the plasma current is not affected by the proposed ES-based VS system and the plasma boundary does not touch the first wall in any of the considered scenarios. These results have been validated by means of both linear and nonlinear simulations.

It is worth to stress once again that the main advantage of the proposed technique resides in the fact that it can be adapted rather easily to different plasma configurations. In fact, this usually requires low or no effort, provided that the considered observer is capable of describing, at least roughly, the unstable dynamic of the plant and that suitable controller gains are chosen. This is not the case for standard VS techniques, which usually need to be tuned on the basis of the specific plasma configuration, a task that usually requires some significant modelling and testing effort. This opens an interesting perspective for the development of model-free VS stabilization techniques. Along this path, a future line of research will attempt to use fully data-driven techniques in order to eliminate even the residual model-dependence embedded in the Kalman filter. Further investigation will also focus on the reduction of the maximum voltage for the switching power supply.

CRediT authorship contribution statement

S. Dubbioso: Conceptualization, Methodology, Writing – original draft, Software. **L.E. di Grazia:** Writing – original draft, Software. **G. De Tommasi:** Conceptualization, Supervision. **M. Mattei:** Writing – review & editing. **A. Mele:** Methodology, Writing – original draft. **A. Pironti:** Writing – review & editing.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: all the authors report financial support was provided by EUROfusion Consortium. Gianmaria De Tommasi, Alfredo Pironti, Massimiliano Mattei report financial support was also provided by Government of Italy Ministry of University and Research.

Data availability

Data will be made available on request.

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